Half-Yield Discharge: Process-Based Predictor of Bankfull Discharge

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Abstract: The river management and restoration community has devoted much effort to predicting the bankfull discharge, Q_{bf}, and associated channel geometry at Q_{bf} for the purposes of channel study, classification, and design. Four types Q_{bf} prediction methods predominate: (1) direct estimation based on field indicators of bankfull stage, (2) downstream hydraulic geometry equations, (3) a flood peak discharge with a specified return interval based on the annual maximum flood series (e.g., the 1.5- to 2-year flood) or regional flood peak statistical relations, and (4) process-based approaches that incorporate the magnitude and frequency of sediment transport such as the most effective discharge Q_{eff}. Process-based Q_{bf} predictors are calculated using sediment transport data from 95 gauged sites across the United States including coarse, bed load–dominated channels and fine, suspended load–dominated channels with drainage areas ranging from 6 to 1.4×10^5 km^2. These values are compared with observations of Q_{bf} made from field measurements of bankfull indicators. It was found that the discharge associated with 50% of cumulative sediment yield based on the flow record—Q_h, the half-yield discharge—predicts Q_{bf} well. When compared with Q_{eff} and the 1.5- and 2-year floods ($Q_{1.5}$ and Q_2), Q_h has the lowest relative error in predicting Q_{bf} for coarse and fine bed sites. Log-log regression models of observed–predicted data pairs indicate that Q_h and $Q_{1.5}$ calculated for fine bed sites are the only Q_{bf} predictor models whose slopes are not significantly different from unity and whose intercepts are not significantly different from zero. The most effective discharge, Q_{eff}, and Q_h both perform well in predicting Q_{bf} in coarse bed sites, followed by $Q_{1.5}$, whereas Q_{eff} uniformly underpredicts Q_{bf} in fine bed sites. The behavior of Q_h, a process-based predictor of Q_{bf}, is characterized to highlight circumstances in which sediment yield analysis may be important in estimating the bankfull discharge. The half-yield discharge represents a novel estimator of Q_{bf} in rivers not previously discussed in this context. **DOI: 10.1061/(ASCE)HY.1943-7900.0001137.**

Introduction

Stable river form over engineering time frames (50–100 years) results from the balance of flow regime, sediment supply, and imposed valley slope with the resisting forces of boundary materials in the bed and banks as well as vegetation (Lane 1954; Schumm and Lichty 1965; Millar 2005). Though no one discharge is entirely responsible for river form, the bankfull discharge, Q_{bf}, defined conceptually herein as the discharge that just fills the channel before spilling on to the floodplain (Wolman and Leopold 1957; Williams 1978), is one of several important channel geometry and design metrics. This stems from its connection with the dominant or channel-forming discharge concept rooted in river regime theory (e.g., Inglis 1947, 1949; Benson and Thomas 1966; Carling 1988; Soar and Thorne 2011) and early floodplain formation and hydraulic geometry (HG) studies (Wolman and Leopold 1957; Leopold and Maddock 1953; Hey and Thorne 1986).

The dominant discharge can be thought of as that which, when held steady, would result in equilibrium channel geometry, planform, and slope under a given sediment supply. It is a conceptual approximation that integrates the effects of flow and sediment regime in rivers (Inglis 1947; Ackers and Charlton 1970). The dominant discharge concept does not apply to all rivers. Stevens et al. (1975), Wolman and Gerson (1978), Pizzuto (1994), and Soar and Thorne (2011) discussed its application. In some cases, Q_{bf} may have the properties of a dominant discharge in alluvial rivers in that sediment transport effectiveness may be at a maximum at bankfull (Wolman and Leopold 1957; Andrews 1980). For this reason, Q_{bf} or a proxy for Q_{bf} such as a flood of a certain return interval are often used as design discharges for channel restoration and management (Hey and Thorne 1986; Shields et al. 2003; Doyle et al. 2007). Doyle et al. (2007) and Soar and Thorne (2011) have excellent reviews of dominant discharge concepts and methods for estimating it for channel design.

Four predominant approaches to estimating the dominant discharge are practiced: (1) direct estimate based on field indicators of bankfull stage, (2) indirect estimate based on a regional downstream hydraulic geometry relation created from reference reaches or scaled from a nearby reference reach, (3) indirect estimate based on a hydrologic metric (generally the 1.5- to 2-year flood: $Q_{1.5}$ and Q_2), and (4) indirect process-based estimate based on effective discharge, Q_{eff}, analysis [herein referred to as magnitude-frequency analysis (MFA)] to calculate the discharge that transports the most sediment over time or another related sediment yield metric: the half-yield discharge, Q_h. The present study compares the indirect methods, Methods 3 (hydrologic-based) and 4 (hydrologic and sediment continuity–based) to Method 1: direct estimation of Q_{bf} with field measurements of bankfull stage indicators.

The term *process-based*, used to describe dominant discharge estimate Method 4, is used in river restoration literature and generally refers to design approaches that consider geomorphic and ecological processes and not simply channel form or the presence of physical aquatic habitat (Shields et al. 2003; Simon et al. 2007; Beechie et al. 2010). Though hydrologic predictors of Q_{bf} represent an important channel forming process—namely the magnitude and frequency of flood flows—the term *process-based predictors* is

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used herein to refer to the fact that Q_{eff} and Q_h incorporate both hydrologic and sediment transport processes to explicitly consider sediment yield in channel form.

As has been previously discussed in depth, all of these approaches have their limitations. Identifying Q_{bf} in the field relies on interpretation of field indicators of bankfull stage, potentially introducing large error in its estimation (Williams 1978; Shields et al. 2003; Navratil et al. 2006; Harman et al. 2008). In some channels, a well-defined floodplain may not exist, or channel disturbance may create incised conditions or ambiguous indicators of bankfull stage negating the validity of using Q_{bf} to estimate the dominant discharge (Doyle et al. 2007). Finally, extrapolating information from a reference reach to a reach of interest (e.g., Rosgen 1997, 2001) may not be appropriate in certain scenarios wherein the reach of interest is unstable or has different forcing and boundary conditions (Wilcock 1997).

Hydrologic predictors of the dominant discharge may suffice for stable channels lacking major anthropogenic influence such as river engineering or hydromodification from flow regulation or land use change in the watershed (Doyle et al. 2007). Multiple studies in coarse bed rivers have found a wide range of return intervals for Q_{bf} (Wolman and Leopold 1957; Andrews 1980; Williams 1978). The median value of the return interval of Q_{bf} from these studies often falls between 1 and 2 years on the maximum annual flood series, whereas the mean value from these studies is often greater than 2 years. This indicates a positively skewed distribution and a larger range of values for the return interval of Q_{bf}, though Castro and Jackson (2001) found that the return interval of Q_{bf} for Pacific Northwest rivers has a mode of 1 year and is negatively skewed. The return interval of Q_{bf} for channels that have adjusted to disturbance by incising and/or widening will generally be greater than 2 years (Doll et al. 2002; Doyle et al. 2007).

Process-based predictors of Q_{bf} involve calculating discharge indices based on a sediment yield curve, which is the product of a sediment transport relation with a representation of the flow frequency distribution. Though this calculation is more involved than other methods, it can provide more information to the channel designer or manager about sediment continuity, an important consideration in channel design and management (Soar and Thorne 2001; Shields et al. 2003; Doyle et al. 2007). The effective discharge is the discharge associated with the maximum value of the sediment yield frequency curve (product of flow frequency distribution and a sediment transport relation) (Wolman and Miller 1960; Andrews 1980) [Fig. 1(a)]. The half-yield discharge is the discharge associated with 50% of cumulative sediment yield over a flow record (Emmett and Wolman 2001; Vogel et al. 2003). It may be calculated from a cumulative sediment yield curve plotted as a function of the sorted flow record (e.g., Biedenharn and Thorne 1994) [Fig. 1(b)].

In general, Q_{eff} predicts Q_{bf} with mixed performance. In coarse bed rivers dominated by bed load sediment transport, Q_{eff} appears to predict Q_{bf} reasonably well (Andrews 1980; Emmett and Wolman 2001; Hassan et al. 2014), or may be much greater than Q_{bf} (Bunte et al. 2014). In fine bed rivers dominated by suspended load sediment transport, Q_{eff} is often much smaller than Q_{bf} especially in flashy systems, depending in part on MFA methods used and channel type (Pickup and Warner 1976; Soar and Thorne 2001; Hassan et al. 2014). The half-yield discharge tends to be larger than Q_{eff}, especially in fine bed, suspended load–dominated rivers (Vogel et al. 2003; Klonsky and Vogel 2011; Sholtes 2015); as such, it may pose a better metric for characterizing the magnitude of transported load in rivers than Q_{eff}.

Previous studies have considered Q_h as a sediment yield metric in rivers (Emmett and Wolman 2001; Vogel et al. 2003; Copeland et al. 2005; Klonsky and Vogel 2011; Hassan et al. 2014). A gray literature report found good agreement between Q_h and Q_{bf} on six sites on the Lower Brazos River, Texas, one of which was used in the present study (Strom and Hosseiny 2013). However, to the authors' knowledge, no other published work has directly evaluated the ability of Q_h to predict Q_{bf} across a wide range of river types and flow regimes.

Copeland et al. (2005) found that the discharge associated with 75% of cumulative sediment yield, Q_{s75}, predicts Q_{bf} well in fine bed rivers; however, they use total suspended load in their estimates, which includes wash load. Wash load comprises silt and clay-sized particles and generally does not form the channel bed of most sand bed streams (Biedenharn and Thorne 1994; Hey 1996). Wash load material can settle out on the floodplain during overbank flows forming cohesive banks, the stability of which influence channel geometry (Millar and Quick 1993; Lauer and Parker 2008). Though important for channel form, the influence of bank stability on channel geometry is not considered in the present study. Here sediment load data of sand-sized material and larger are used to calculate Q_{eff} and Q_h and compare them with $Q_{1.5}$ and Q_2 in their ability to predict Q_{bf} as estimated from bankfull stage field indicators. Sediment transport data used in this analysis for fine bed rivers are the fraction of the suspended load ≥ 0.0625 mm as measured and reported by the U.S. Geological Survey (2016b), and for coarse bed rivers is the bed load measured using Helley-Smith bed load samplers. These metrics are calculated using a national database of fine and coarse bed sites and their predictive ability is compared with hydrologic metrics.

This study involves three methods for estimating Q_{bf}: (1) direct estimation using measurements of field indicators of bankfull stage

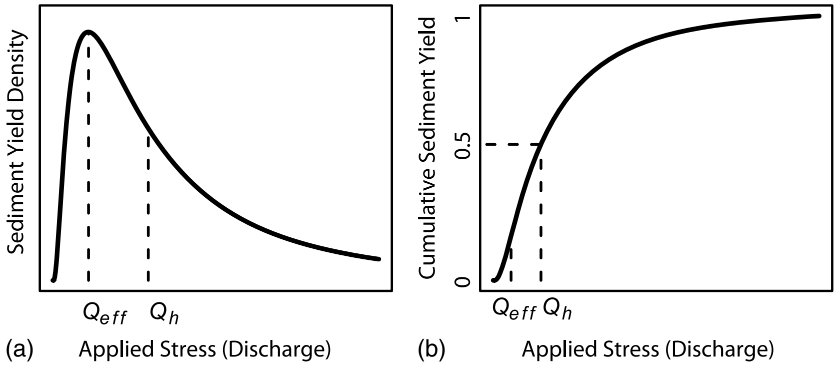

Fig. 1. Conceptual diagram of process-based Q_{bf} predictors, Q_{eff} and Q_h, based on (a) the sediment yield probability density curve; (b) the cumulative sediment yield curve

at a site with an established stage-discharge rating curve, (2) indirect estimation using a flood peak discharge with a specified return interval based on the annual maximum peak discharge series at a gauged site ($Q_{1.5}$ and Q_2), and (3) indirect estimation using process-based discharge indices based on MFA (Q_{eff} and Q_h). A goodness-of-fit (GOF) analysis is conducted to compare direct estimates of Q_{bf} with predictors based on annual flood series and sediment yield MFA and consider factors influencing their accuracy. The paper concludes with practical recommendations regarding the use and accuracy of Q_h in predicting Q_{bf}.

Data and Methods

Site Data

Sites used in this study are located across the conterminous United States and Puerto Rico and include 58 fine bed sites and 37 coarse bed sites for which estimates of Q_{bf} were available near a gauge with a long-term record and ≥15 paired sediment load-discharge measurements (Fig. 2). As described in further detail subsequently, sites were discriminated into categories of fine and coarse bed, defined as channels with median bed grain sizes that are generally ≤1 mm and ≥4 mm. These sites were selected because the majority have been previously published in MFA studies or are located along the same river as previously published sites (Tables S1 and S2). Other sites were brought in to this study to augment those used in previous MFA studies. This allowed incorporation of sites with smaller drainage areas and expansion of the number of sites with the fraction of suspended sand evaluated in suspended sediment concentration measurements. Only alluvial rivers (mobile bed and banks) that are in dynamic equilibrium with the drivers of flow and sediment supply were included, meaning measured channel properties are likely to have a stable mean value over an engineering time frame (50–100 years). Anthropogenic impacts such as flow regulation, channelization, and land use change can result in transient influences on channel form. Aerial photograph reconnaissance was used, as well as USGS gauge information as a rough method to verify that these sites are located on a river that is minimally influenced by flow regulation or channelization. Fine bed sites are scattered geographically and have a range of flow regimes, whereas coarse bed sites are clustered in the U.S. Rocky Mountain and Northwest regions due to lack of concurrent bed load and stream gauge data availability elsewhere. Flow regimes for coarse bed sites are mostly snowmelt dominated. Summary information for sites used in the present study can be found in Fig. S2 and Tables S1 and S2.

Bankfull Discharge Estimation

Two primary methods are used to generate the field measurement–based estimates of Q_{bf}: (1) previously published estimates made directly in the field from surveyed elevations of the transition from bank to floodplain along a reach and then extrapolated by elevation to a nearby stream gauge with an established stage-discharge rating curve and benchmark (direct method) (King et al. 2004), and (2) estimates derived from at-a-station hydraulic geometry relationships using USGS discharge field measurement data based on methods described by Williams (1978) (hydraulic geometry method). Regional downstream hydraulic geometry statistical relations were used to estimate Q_{bf} for four coarse bed sites (Foster 2012). The majority of bankfull discharge estimates at coarse bed sites are derived from the direct method (28 out of 37) and the majority of estimates for fine bed sites are derived from the at-a-station hydraulic geometry method (44 out of 58, Tables S1 and S2). Regardless of the method used, estimating bankfull discharge based on field indicators is inherently uncertain (Johnson and Heil 1996; Navratil et al. 2006; Harman et al. 2008), and this uncertainty likely adds variability to the relationships between observations and predictions of Q_{bf}.

Williams (1978) describes the following at-a-station hydraulic geometry relationships as useful for determining Q_{bf} based on identifying the discharge associated with (1) the minimum value of the width-to-depth ratio [Fig. 3(a)], (2) a break in slope from steeper to less steep in the stage-discharge relationship [Fig. 3(b)], (3) a discontinuity or vertical jump in the top width–discharge relationship [Fig. 3(c)], and (4) a discontinuity or horizontal jump in the top width–cross sectional area relationship [Fig. 3(d)]. All approaches were used to evaluate Q_{bf}, and Methods 2 and 3 were found to

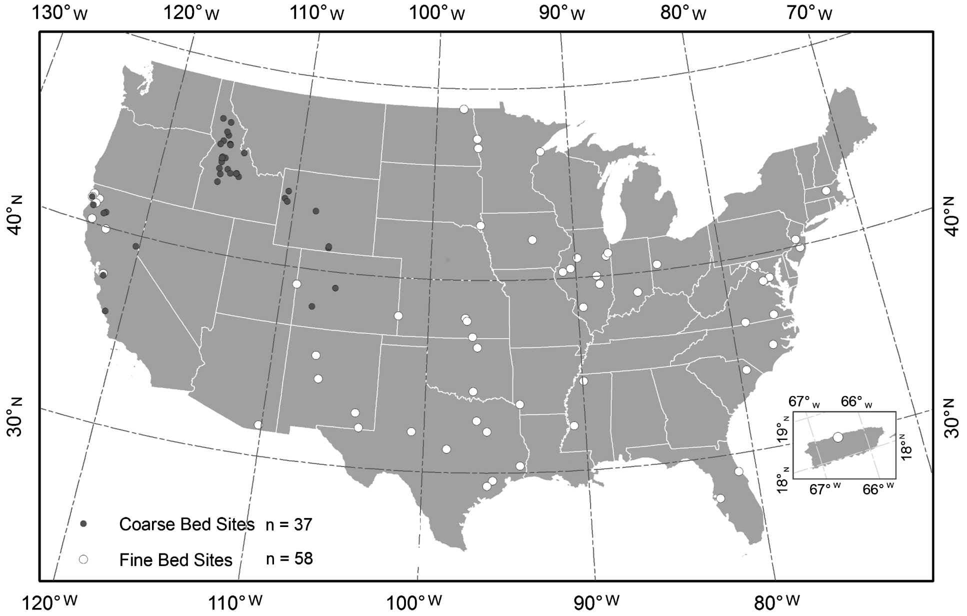

Fig. 2. Site map of fine bed and coarse bed sites utilized in this study

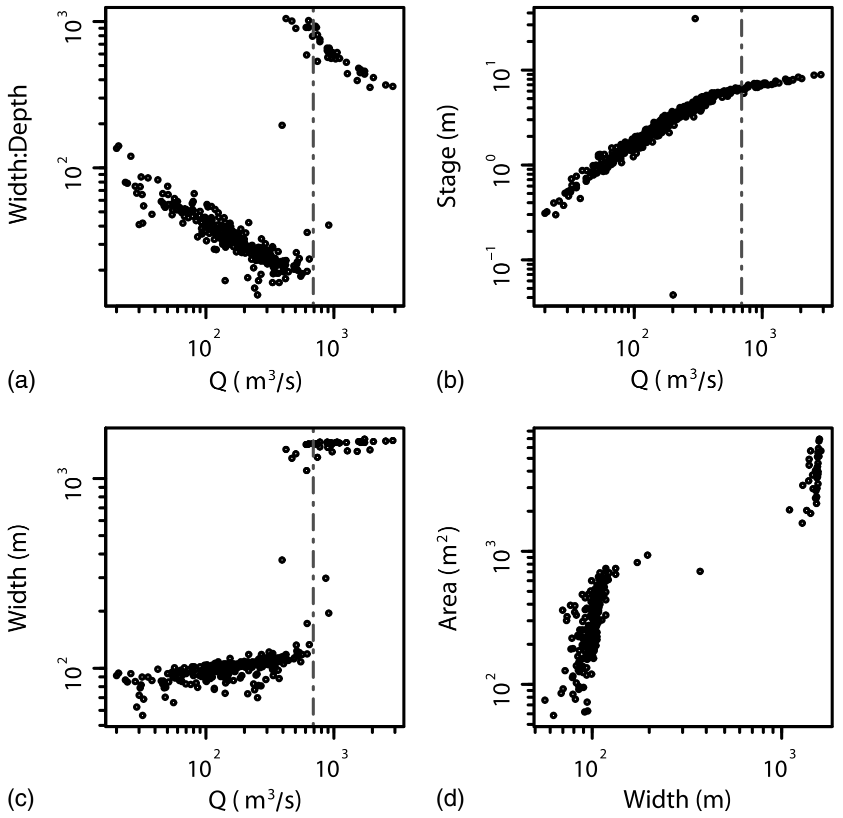

Fig. 3. Example of Q_{bf} determination made from USGS field discharge measurements made on the Pee Dee River at Pee Dee, South Carolina (USGS Gauge 02131000); dashed vertical line indicates Q_{bf}; (a) minimum value in the Q–W:D relationship; (b) change in slope in stage-discharge relationship; (c) abrupt increase in width all at approximately the same discharge value); (d) cross section area to channel top width relation also shows a break around Q_{bf}

provide the clearest indication of Q_{bf} at most sites if such a break existed in the available data.

In some cases, estimates of Q_{bf} based on the at-a-station hydraulic geometry relationships coincided with sites that had previously published, direct estimates of Q_{bf} ($n = 18$). Field measurements for both methods were collected along the same reach of channel, though they may not coincide in time and space, introducing some error. These concurrent direct and hydraulic geometry-based estimates were used to evaluate the accuracy and bias of the at-a-station hydraulic geometry approach to estimating Q_{bf}. A 1:1 plot of observed (published, direct) and estimated (hydraulic geometry) data shows little bias in the estimation method and a reasonably good fit with observations [Fig. 4(a)]. The mean percent error (MPE) is $+5\%$ and mean absolute percent error (MAPE) 50% [Fig. 4(b)]. Percent error is calculated as $[(Q_{\mathrm{bf,meas}} - Q_{\mathrm{bf,est}})/Q_{\mathrm{bf,meas}}] \times 100$. This indicates that the at-a-station hydraulic geometry method for estimating Q_{bf} using USGS field measurements is only slightly positively biased and reasonably accurate.

Considering the influence of the two Q_{bf} estimation methods on the results, first the means of the distribution of Q_{bf} estimates were compared using Welch's two-sample t-test of mean values of Q_{bf} for each method within each stream type. This was not significant [$t(13.8) = 1.3$, $p = 0.21$ for fine bed sites and $\mathrm{t}(32.6) = 0.74$, $p = 0.46$ for coarse bed sites]. Boxplots of these values indicate substantial overlap [Fig. 4(c)]. An analysis of covariance (ANCOVA) considering the influence of Q_{bf} estimation method on the relationship between Q_{bf} and Q_h indicates that the estimation method does not significantly affect the slope [$F(1{,}91) = 1.86$, $p = 0.18$] or the intercept [$F(1{,}92) = 1.07$, $p = 0.30$] of the regression line fit to log-transformed $Q_{\mathrm{bf}} - Q_h$ data pairs segregated by Q_{bf} estimation method. This is evident in the overlap of these regression lines [Fig. 4(d)]. Furthermore, confidence bands for both Q_{bf} estimation methods overlap the 1:1 line [Fig. 4(d)]. Log-transformation was necessary given the range of values spanning orders of magnitude to reduce the leverage of high value data pairs as well as heteroscedasticity.

Bankfull Discharge Prediction

Two different types of predictors are compared: hydrology- and process-based. The hydrologic predictors are calculated using the annual maximum flood series available on the USGS National Water Inventory Service (NWIS) online database (USGS 2016a). The $Q_{1.5}$ and Q_2 floods are estimated using the Weibull plotting position, $T = (n + 1)/m$, and linear interpolation, where T is return interval (years), m is the rank of the event (with 1 being the largest), and n is the number of events on record.

Process-based predictors of Q_{bf}, such as sediment yield metrics Q_{eff} or Q_h, rely on the product of a flow frequency distribution [probability density function (PDF)] or histogram with a sediment rating curve to create a sediment yield density curve for Q_{eff} [Fig. 1(a)], or in the case of Q_h, a cumulative sediment yield curve [Fig. 1(b)]. The cumulative sediment yield curve can be thought of as the transformation of a flow duration curve into a sediment yield duration curve using a sediment rating curve (USACE 1989). Biedenharn and Thorne (1994) demonstrate this method with flow and sediment transport data on the Mississippi River.

The product of the empirical PDF of average daily flow with a sediment rating curve is used to calculate Q_{eff}. The flow PDF is

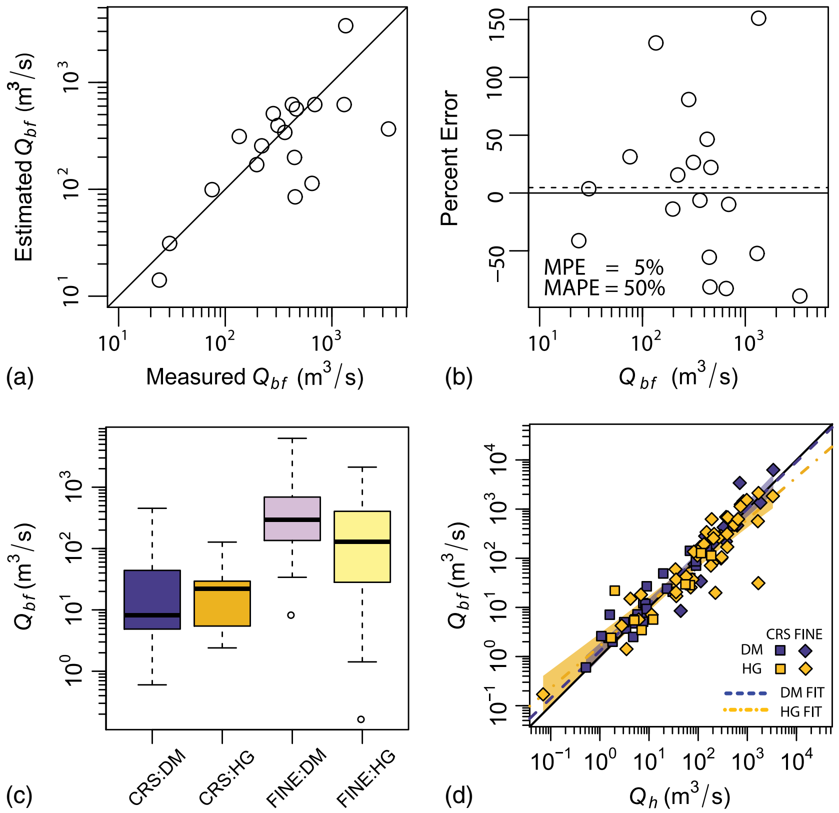

Fig. 4. (a) Comparison between estimates of Q_{bf} derived from at-a-station hydraulic geometry relationships made from USGS field measurements and estimates of Q_{bf} directly determined in the field; (b) individual values of the percent error for sites from (a), MPE and MAPE values are provided along with a dashed line indicating the value of the MPE (5%); (c) boxplots of estimated Q_{bf} values by stream type and measurement type; (d) log-log regressions fit to $Q_{\mathrm{bf}} - Q_h$ data pairs grouped by Q_{bf} measurement type as part of analysis of covariance on influence of Q_{bf} measurement type: direct (DM) and indirect or at-a-station HG methods; shaded regions are confidence bands for both measurement types, which overlap the 1:1 line

estimated by numerically deriving a smoothed, empirical cumulative distribution function (CDF) (e.g., Orndorff and Whiting 1999; Sholtes 2015). The discharge value at the peak of this sediment yield density curve is Q_{eff} [Fig. 1(a)]. The half-yield discharge can be calculated directly from the sorted sediment yield record by transforming the gauged flow series into a sediment yield series using a sediment rating curve or a calibrated sediment transport model. Alternatively, if using a scaled or regional flow duration curve, Q_h can be calculated from the cumulative integral of the sediment yield density curve [Fig. 1(b)].

The sediment rating curve is a back-transformed log-log regression equation of the form $Q_s = \alpha Q^{\beta}$, created from instantaneous discharge and sediment transport data pairs. The rlm() function in the *MASS* package (Venables and Ripley 2002) in R (R Core Team 2013) is used, which is a robust linear regression method that is less sensitive to outliers than ordinary least-squares (OLS) regression. Transformation bias is corrected using the bias correction factor discussed by Ferguson (1986). All log-log slopes (β values) are significant using an approximation of normality test calculated with the lmRob() function in the robust package (Wang et al. 2014) (maximum $p = 0.014$). Multiple R^2 values from these robust linear models are calculated from the weighted residuals and ranged from 0.21 to 0.79 (median = 0.57); however, 70% of these values are greater than or equal to 0.5. These multiple R^2 values are lower than the conventional R^2 values derived from OLS regression. Values of R^2 from OLS regression range from 0.22 to 0.96 (median = 0.74) with 84% ≥0.5.

Direct sediment transport measurements are available for coarse bed rivers in units of mass/time. Helley-Smith bed load sampler data were only used because these are the most widely available data. Though it is a widely used, the Helley-Smith sampler has been criticized for oversampling sand-sized material (down to ≈0.25 mm) at lower flows and undersampling coarse fraction of bed load (large gravel and cobble-sized material) (Bunte and Abt 2009). Vericat et al. (2006) found that 76-mm Helley-Smith samplers, which were used to sample the bed load at many of the sites used in the present study, tended to sample smaller particles and less bed material compared with the larger 152-mm Helley-Smith sampler. This sampling bias represents a limitation of the bed load data set used in this study. However, given the pervasive use of the Helley-Smith sampler within the United States, no other equivalently large data set for bed load exists for this type of analysis. Bed load data come from a wide variety of sources listed in Table S1.

Sediment load for fine bed sites is calculated as the product of the sand fraction (≥0.0625 mm) of the measured suspended sediment concentration with a concurrent instantaneous discharge measurement to produce sediment load in units of mass/time. Suspended load data for all fine bed sites come from the USGS Sediment Data Portal (USGS 2016b), an online database of suspended sediment measurements for sites across the United States and its territories.

Because the calculations of sediment yield for fine bed sites are solely based on suspended sand load and neglect the unmeasured bed load, these likely underestimate cumulative bed material yield. Nash (1994) and others argue that the value of the rating curve exponent is not greatly impacted by inclusion of bed load in sand bed streams, and therefore sediment yield metrics such as Q_{eff} and Q_h are not affected because they are only sensitive to this parameter in the sediment rating curve (Benson and Thomas 1966; Nolan et al. 1987; Biedenharn and Thorne 1994).

Using published USGS bed load and suspended sand data sets collected concurrently at several sand bed sites (Elliot and Anders 2005), Michels-Boyce ("A comparison between rating curves generated from total and suspended bed material loads in sand bed channels," unpublished report) found a similar outcome. That is, the value of the sediment rating curve exponent, β, did not vary much when bed load data were added to suspended sand data and a log-log regression conducted. This means that β values are not likely to significantly change for fine bed sites if bed load data were added to the regression model.

This paper's sediment yield calculations for coarse bed sites only consider measured bed load and not suspended bed material load (sand) as well. Though suspended sand may comprise a significant portion of total sediment yield in some coarse bed rivers, it is the transport of the coarse bed material particles that comprise the channel boundary that is of interest herein. However, unlike the lack of influence of excluding bed load in fine bed rivers on the sediment rating curve, excluding suspended sand load data in coarse bed sites would likely result in a sediment rating curve with a smaller β value. This is because more sediment would be in transport at lower flows. This would tend to reduce the estimate of Q_h and Q_{eff} for these sites. Indeed, Bunte et al. (2014) find much larger values of β for bed load transport in coarse bed rivers using bed load trap samplers, which do not capture sand-sized material and are able to sample larger sediment. Their bed load rating curves resulted in very large estimates of Q_{eff}. Hassan et al. (2014) modeled sand and gravel transport in an effective discharge study in mixed-load streams. They found that streams with significant sand present in the bed and in transport had much lower values of Q_{eff} ($Q_{\mathrm{eff}} < Q_{\mathrm{bf}}$) relative to those with less sand. This type of magnitude-frequency analysis captures sediment transport processes and not necessarily channel-forming processes. By representing sand bed load, armor layer movement may be underrepresented or given less weight in overall sediment yield (an acknowledged weakness of the Helley-Smith bed load sampler as well).

Though it is a simplification, representing the bed load transport only in coarse bed streams (which does include sand-sized material moving along the bed as captured by Helley-Smith bed load samplers) characterizes the magnitude and frequency of channel boundary transport reasonably well in these supply-limited environments. These bed load transport data are imperfect in characterizing bed material load in these rivers, and this paper explores how this might influence the interpretation of the results in the subsequent discussion.

Predictor Goodness-of-Fit and Uncertainty Analysis

The GOF of the Q_{bf} predictors was measured using several metrics describing relative error between observed and predicted values of Q_{bf} as well as relative variability in the difference between observations and predictions. To estimate relative error with respect to the 1:1 line, the Theil (1958) measure of association was used, which is a measure of normalized distance between predicted and observed points, and unlike the correlation coefficient, also accounts for how close to the 1:1 line the predictions are. This metric is recommended by Smith and Rose (1995) for GOF analysis. Theil's measure of association, U, is defined as follows:

$$U = \frac{\sqrt{\frac{1}{n}\sum (y_i - \hat{y}_i)^2}}{\sqrt{\frac{1}{n}\sum y_i^2} + \sqrt{\frac{1}{n}\sum \hat{y}_i^2}} \tag{1}$$

where y_i = observed value; and $\hat{y}_i$ = predicted value. Smaller values of U indicate a better fit to the 1:1 line. The root-mean-square deviation (RMSD) between predicted and observed values recommended as a GOF metric by Pineiro et al. (2008) was also considered. The RMSD has the same units as the variables ($\mathrm{m^3/s}$) and represents the mean deviation across all observations. This means it is sensitive to large differences between larger observations

$$\mathrm{RMSD} = \sqrt{\frac{1}{1-n}\sum (y_i - \hat{y}_i)^2} \tag{2}$$

The standard error (SE) of the residuals between observed and predicted values assuming a 1:1 relationship is calculated as

$$\mathrm{SE} = \frac{s(y_i - \hat{y}_i)}{\sqrt{n}} \tag{3}$$

where $s(y_i - \hat{y}_i)$ = standard deviation of the differences between observed and predicted values. This metric describes the level of uncertainty or accuracy associated with a particular predictor averaged over all values, meaning it is likely an overestimate of accuracy for small values of predicted Q_{bf} and underestimate of accuracy for large values of predicted Q_{bf}.

Statistical tests between regression lines are used to fit to log-transformed, observed-predicted data pairs to determine whether different predictors result in significantly different regression lines and to determine if the slope and intercepts of these lines are significantly different from unity and zero, respectively. Log transformation is used because the residual error is heteroscedastic, which is expected when the observations span several orders of magnitude. The data were checked for outliers by conducting a Bonferroni-corrected t-test with $\alpha = 0.05$ on studentized residuals from the log-log models (Cook and Weisberg 1982) as implemented in the outlierTest() function in the car package in R (Fox and Weisberg 2011). This resulted in the removal of two sites within each river type from the original data set.

To estimate the uncertainty associated with predicting Q_{bf} using one of the predictors examined, the mean of the back-transformed, standard error of the prediction (SEP) as a percentage as well as approximate 95% prediction limits were calculated as discussed in the supplemental materials. The SEP metric is similar to that reported by the USGS regarding flood peak discharge estimates made from regional regression equations (Ries et al. 2002).

This goodness-of-fit analysis compares the relative accuracy, bias, and uncertainty associated with each Q_{bf} predictor as evaluated by each of these GOF metrics. The best-performing predictor will have the least overall error or best accuracy in predicting Q_{bf} (Theil's U, RMSD, SE), it will be the least biased (visual and regression model), it will not be significantly different from the 1:1 line (regression model), and it will also have the least uncertainty (narrowest confidence interval). Not all of these performance indicators may hold true for a single Q_{bf} predictor. Therefore, predictor performance is evaluated based on the preponderance of evidence provided by the GOF metrics.

Results

The performance of hydrologic ($Q_{1.5}$ and Q_2) and process-based (Q_{eff} and Q_h) predictors of Q_{bf} are compared using GOF metrics

Table 1. Bankfull Discharge Predictor Goodness of Fit and Regression Metrics

Q_{bf} predictor	RMSD	U	SE	Line slope	Linear R^2	Log slope[a]	Log slope p	Log R^2	Intercept[b]	Intercept p[c]
					Coarse bed sites					
Q_{eff}	30	0.13	5	**0.94**	0.92	0.74	0.001	0.74	2.74	4.0×10^{-05}
Q_h	**25**	**0.10**	**4**	0.88	**0.96**	**0.85**	0.013	**0.86**	1.66	0.007
$Q_{1.5}$	59	0.23	9	0.75	0.79	0.81	0.004	0.83	1.66	0.02
Q_2	82	0.28	13	0.62	0.78	0.80	0.004	0.81	1.41	**0.14**
					Fine bed sites					
Q_{eff}	850	0.55	104	1.78	0.45	0.67	7.0×10^{-06}	0.64	13.72	2.7×10^{-13}
Q_h	**650**	**0.33**	**85**	1.07	**0.58**	**0.92**	**0.21**	0.79	1.34	**0.39**
$Q_{1.5}$	730	0.40	95	1.25	0.49	**1.08**	**0.25**	0.81	0.48	**0.06**
Q_2	690	**0.33**	90	**0.95**	0.52	1.11	**0.11**	**0.83**	0.29	2.3×10^{-03}

Note: Best or values indicating good fit are in bold.

[a] Log slope p refers to the p-value associated with a t-test of the observed-predicted log-linear regression model diverging from unity with p-values ≤ 0.05 demonstrating that the slope of the regression line is significantly different from unity with a probability of 95%.

[b] Intercept values are back-transformed.

[c] Intercept p-values indicate whether the intercept is significantly different from zero.

and linear regression models between observed and predicted values for coarse and fine bed sites (Table 1). First, metrics that estimate the relative error and variability between observed and predicted values of Q_{bf} are compared. The half-yield discharge exhibited the lowest values of U, RMSD, and SE (and hence best prediction by these criteria) for coarse bed sites followed closely by Q_{eff} and then $Q_{1.5}$. In fine bed sites Q_h also tended to have the lowest values of these GOF metrics, though it shares the lowest value of U with Q_2. However, on the whole, Q_2 overpredicts Q_{bf} for fine bed sites. The 1.5- and 2-year recurrence interval flood peak discharge values followed Q_h in overall performance with these GOF metrics in fine bed sites, with Q_{eff} performing the poorest overall.

Next, the log-log regression lines fitted to the predicted and observed Q_{bf} values are considered (Fig. 5, Table 1). This allows for rapid visual determination of bias and error. The discharges associated with the median value (Q_h) and interquartile range (IQR) of cumulative sediment yields (Q_{s25} and Q_{s75}) are calculated to compare their relationships with Q_{bf} as well [Figs. 5(a and b)]. The log-log regression line for the $Q_{\mathrm{bf}} - Q_h$ relationship falls nearly on top of the 1:1 line for fine bed sites, and is bracketed by Q_{s25} and Q_{s75} [Figs. 5(a and b)]. The effective discharge tends to underpredict Q_{bf} for both river types, but performs nearly as well as Q_h for coarse bed sites [Figs. 5(c and d)]. Finally, the hydrologic predictors perform nearly as well as the process-based predictors for coarse bed sites, while tending to overpredict Q_{bf} for fine bed sites [Figs. 5(e and f)]. The linear R^2 value associated with the fine bed Q_h model is the highest of all predictors (0.58), and the log-log R^2 value (0.79) falls very near the greatest R^2 value associated with the Q_2 predictor (0.83). In coarse bed sites, the linear R^2 value for Q_h is second highest (0.88) compared with Q_{eff} (0.94); whereas in fine bed sites the log-log R^2 value is greatest for Q_h (0.86) followed by $Q_{1.5}$ and Q_2 (0.83 and 0.81). Some of the variability in the observed-predicted relationships likely also comes from the inherent uncertainty in estimating Q_{bf} in the field.

Regression model significance tests indicate that all log-transformed observed–predicted Q_{bf} relationships have slopes that are significantly less than unity for coarse bed sites (maximum $p = 0.013$ for Q_h, Table 1). As reported previously, log-log slopes range from 0.85 for Q_h to 0.74 for Q_{eff}. For fine bed sites, the log-log slope of the Q_h and $Q_{1.5}$ models are equally close to unity and are the only models with slopes not significantly different from unity ($p = 0.21$ and 0.25, respectively). The intercepts of these models for fine bed sites are also the only ones not significantly different from zero ($p = 0.39$ for Q_h and 0.06 for $Q_{1.5}$). An ANCOVA analysis indicates that with the exception of the Q_{eff}–Q_{bf} relationship for fine bed sites, none of the slopes of the observed–predicted Q_{bf} models are significantly different from one another within each bed material stream type.

Uncertainty analysis metrics were calculated to characterize the relative accuracy of each Q_{bf} predictor (Table 2). The approximate prediction interval of each predictor regression model is reported as a single log-transformed value. See supplemental material section "Bankfull Discharge Predictor Uncertainty Analysis," Eqs. (S2) and (S3) for details on back-transforming logarithmic prediction limits. Uncertainty estimates of each predictor generally follow the trends in GOF metrics described previously. The average percent standard error of the prediction is lowest for Q_h for coarse bed sites and is one point larger that that of $Q_{1.5}$ and Q_2 for fine bed sites. Values of the SEP are just over 100% for all Q_{bf} predictors.

Discussion

Performance of Bankfull Discharge Predictors

Using a nationwide data set of combined flow and sediment transport data, it was found that Q_h is a good predictor of Q_{bf} for both types of sites. It performs similarly to Q_{eff} in coarse bed sites and similar to $Q_{1.5}$ in fine bed sites. The effective discharge performed the least well in predicting Q_{bf} over all for fine bed sites. Though the regression lines for all predictor metrics in coarse bed sites are significantly different from unity, indicating an imperfect predictive fit, other GOF metrics were the lowest for Q_h followed by Q_{eff} for these sites. Confidence bands for each regression line are plotted in Fig. 5, indicating reasonable overlap with the 1:1 line. Because no one predictor performs the best over all GOF metrics, it cannot be definitely concluded that one predictor is superior to the others. Nevertheless, the GOF criteria selected indicate that Q_h performs as well as and perhaps slightly better than Q_{eff}, an oft-cited predictor of dominant discharge, in coarse bed sites and slightly better than $Q_{1.5}$ in fine bed sites.

Hydrologic metrics based on annual maximum flow series such as $Q_{1.5}$ and Q_2 have been reported to approximate Q_{bf} fairly well in coarse and fine bed streams, though considerable variability in the return interval of Q_{bf} exists (Wolman and Leopold 1957; Leopold and Dunne 1978; Williams 1978; Castro and Jackson 2001). Little guidance exists as to when these hydrologic metrics may be reasonable predictors of Q_{bf}, though from this study it appears that Q_{bf} prediction from hydrologic metrics should be limited to the

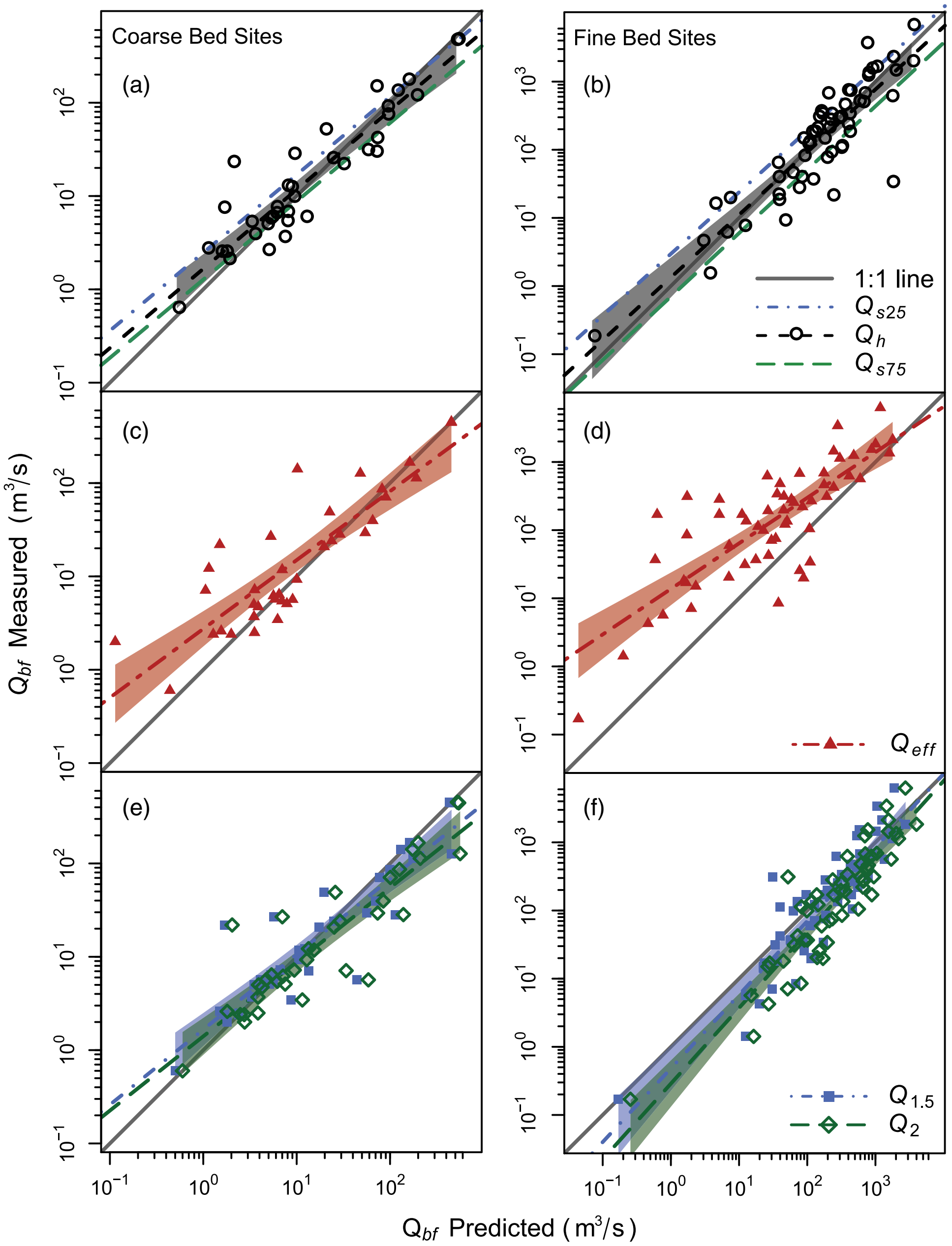

Fig. 5. Log-log plots with log-log regression lines between hydrology- and process-based predicted values and measurements of Q_{bf} made from bankfull field indicators; values of Q_{s25}, Q_h, and Q_{s75} represent the discharges associated with 25, 50, and 75% of cumulative sediment yield, respectively

$Q_{1.5}$ in coarse and fine bed streams because Q_2 tends to overpredict Q_{bf} for both types of sites. However, generalizations made regarding coarse bed sites are limited due to the relatively narrow geographic scope of coarse bed site data, which are dominated by sites located in the U.S. Rocky Mountain region. Estimating these hydrologic Q_{bf} predictors is much easier than process-based predictors because they require only an annual maximum flood peak series from a nearby stream gauge or readily available regional peak discharge regression equations.

Table 2. Bankfull Discharge Predictor Uncertainty Analysis Metrics

Q_{bf} predictor	SE prediction (%)		Median log predictor interval[a] log (m³/s)	
	Coarse	Fine	Coarse	Fine
Q_{eff}	109	108	1.72	2.44
Q_h	104	104	1.26	1.85
$Q_{1.5}$	105	103	1.41	1.77
Q_2	105	103	1.48	1.69

[a] Prediction intervals are given as logarithmic values; see supplemental material section "Bankfull Discharge Predictor Uncertainty Analysis," Eqs. (S2) and (S3) for details on back-transformation.

Comparisons of Q_{eff} with Q_{bf} have produced mixed results. Some researchers have found a close 1:1 relationship between the two (e.g., Andrews 1980; Emmett and Wolman 2001; Torizzo and Pitlick 2004), whereas others have found an inconsistent relationship, with Q_{eff} falling below the value of Q_{bf} (Pickup and Warner 1976; Soar and Thorne 2001; Hassan et al. 2014) or well above it (Bunte et al. 2014). Judging from the literature, it seems that Q_{eff} may approximate Q_{bf} in coarse, bed load–dominated rivers, though Bunte et al. (2014) argue that this is an artifact of bed load sampling method. In their study of the most effective discharges in mountain streams in British Columbia, Canada, Hassan et al. (2014) modeled sand and gravel bed load transport using the Wilcock and Kenworthy (2002) relation. They found that in sites with more sand present in the bed, smaller discharges were relatively more effective, sand transport dominated the total sediment yield, and $Q_{\mathrm{eff}} < Q_{\mathrm{bf}}$. In sites with less sand present, Q_{eff} better approximated or even exceeded Q_{bf}. Infrequently mobile gravel and cobble dominated the beds in these sites. The effective discharge better approximated Q_{bf} for the coarse bed sites used in this study as well. Bunte et al. (2014) found that $Q_{\mathrm{eff}} \gg Q_{\mathrm{bf}}$ in small, snowmelt-driven coarse bed streams. Sand was not included in the bed load measurements used in this study, which involved bed load traps and extended sampling times, resulting in very large β values compared with sediment rating curves generated from Helley-Smith data. Soar and Thorne (2001) found that $Q_{\mathrm{eff}} \approx Q_{\mathrm{bf}}$ in fine bed rivers with low flow variability, but that Q_{eff} became increasingly smaller relative to Q_{bf} as flow variability increased.

This may indicate that there are two end members of effective flow frequencies. Either Q_{eff} is a very frequent flow that is less than bankfull in sand bed streams with more variable flow regimes and small β values, or it is a vanishingly infrequent flow equal to or much greater than bankfull in infrequently mobile, coarse bed streams with large β values and low flow variability. Low values of β (e.g., 1 to 2.5) are most often associated with fine bed rivers (Nash 1994; Syvitski et al. 2000).

The half-yield discharge has not received much attention regarding its ability to predict Q_{bf} or its use as a design discharge. Emmett and Wolman (2001) calculate it in coarse bed streams and note that it tends to approximate Q_{eff} and Q_{bf}. Vogel et al. (2003) derive closed-form solutions for Q_h based on a power-law sediment rating curve and lognormal flow distribution. They along with Klonsky and Vogel (2011) compare it with Q_{eff} in suspended load–dominated rivers and argue that it may be a better discharge index for characterizing suspended and dissolved river loads because Q_{eff} tends to be a relatively frequent discharge in suspended and wash load–dominated rivers. Copeland et al. (2005) report that the discharge associated with the 75th percentile of cumulative sediment yield, Q_{s75}, best predicts Q_{bf}. However, like Vogel et al. (2003), they used total suspended load data, which include wash load and not simply suspended sand load (suspended bed material). This resulted in them predicting a larger suspended sediment load for a given discharge, especially at lower flow rates, likely reducing the value of β and shifting upward the cumulative sediment yield percentile most closely associated with bankfull discharge.

This study extends and adds on previous work regarding Q_h by considering its predictive ability of Q_{bf} over a wide range of river sites across both coarse and fine bed rivers using suspended sand data in fine bed rivers and bed load data in coarse bed rivers.

Cumulative Sediment Yield

To explore the relationship between cumulative sediment yield and various Q_{bf} predictors, consider the cumulative sediment yield percentiles for Q_{bf} and Q_{eff} for the coarse and fine bed streams used in this study, noting that this value is by definition 50% for Q_h. The median value of cumulative sediment yield percentage at Q_{bf} is 50% for fine bed sites (33 to 70% IQR) and 54% for coarse bed sites (35 to 67% IQR) (Fig. 6). The average value of cumulative sediment yield at Q_{bf} is also approximately 50% for both fine and coarse bed sites. Median values of cumulative percent yield for Q_{eff} are much lower than those of Q_{bf} for fine bed sites (13%) and slightly lower for coarse bed sites (42%).

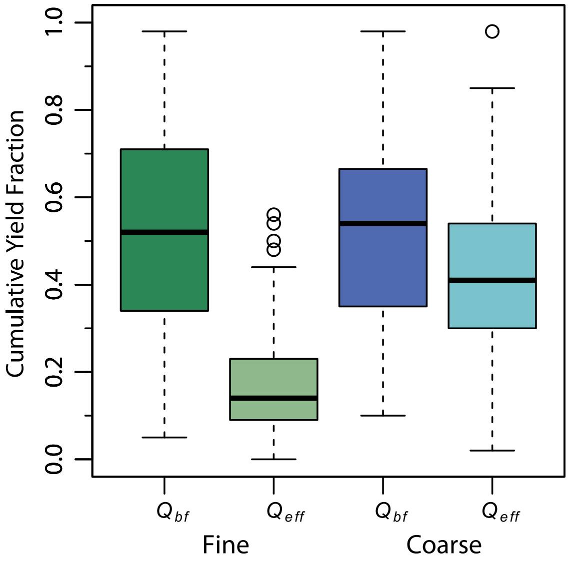

Fig. 6. Box and whisker plots of the fraction of cumulative sediment yield evaluated at Q_{bf} and Q_{eff} for fine and coarse bed sites; whiskers extend to the most extreme data point that is no more than 1.5 times the interquartile range from either the first or third quartile

The question stands: Why does Q_h predict Q_{bf} well? Choosing 50% as a value of cumulative sediment yield for Q_{bf} is arbitrary—rivers are not concerned with medians—though, as discussed, this value matches the median value of cumulative sediment yield at Q_{bf} for fine bed sites and is very close for coarse bed sites (Fig. 6). Many theories exist regarding why Q_{eff} approximates Q_{bf} well (Wolman and Miller 1960; Carling 1988; Hey 1996; Soar and Thorne 2011). The effective discharge maximizes the magnitude and frequency of sediment transport over all discharges. In bed load–dominated rivers, flows near bankfull tend to also be frequent and competent enough to meet this criterion (Torizzo and Pitlick 2004). Unlike Q_{eff}, little theoretical argument can be made for the discharge associated with 50% of cumulative sediment yield approximating Q_{bf}. However, the same can be said of $Q_{1.5}$, which enjoys wide acceptance as a reasonable predictor of dominant and/or bankfull discharge. Therefore, Q_h may also be thought of as an index discharge of intermediate magnitude, derived from process-based sediment yield analysis, which predicts Q_{bf} well.

The half-yield discharge is nearly always larger than Q_{eff}, especially in fine bed rivers, making it a potentially more accurate predictor of Q_{bf} for these systems. It is a robust predictor of Q_{bf} in these rivers because it performs well across a wide range of physiographic regions (Figs. 2 and 5). Additionally, calculating Q_h does not suffer from the sensitivity Q_{eff} has to the flow frequency distribution estimation method such a histogram bin width selection (Soar and Thorne 2001). The ability of Q_h to predict Q_{bf} is a novel finding and an argument for process-based methods for channel design for these sites.

Using Bankfull Discharge Predictors

Calculating process-based Q_{bf} predictors is much more involved than calculating hydrologic predictors. However, because they incorporate physical representations of river hydrology, hydraulics, and morphology, they can provide more insight into the influence of process on channel form (Simon et al. 2011) and even allow for prediction of channel response to hydrologic changes (Tilleard 1999). Based on previous work evaluating the utility of Q_{eff} in predicting Q_{bf} and on the findings of this study, it is reaffirmed that Q_{eff} is a good predictor of Q_{bf} in coarse bed streams. Acknowledging variability in reported values of flood recurrence intervals for Q_{bf}, it is also reaffirmed that $Q_{1.5}$ predicts Q_{bf} fairly accurately as well (Williams 1978; Leopold and Dunne 1978; Castro and Jackson 2001; Emmett and Wolman 2001). Doyle et al. (2007) provides criticisms and qualifications for the use of $Q_{1.5}$ or other hydrologic predictors of Q_{bf}, especially in unstable or urban channels. Based on the results from this statistical analysis comparing measured and predicted values of Q_{bf}, using Q_h to predict Q_{bf} in suspended load–dominated rivers with fine beds is generally more accurate than Q_{eff} and offers an improvement in predicting Q_{bf} over $Q_{1.5}$. Also, Q_h predicts Q_{bf} as well as and perhaps slightly better than Q_{eff} and $Q_{1.5}$ in coarse bed sites, based on the limited geographic scope of these sites.

Barry et al. (2008) demonstrate that the Q_{eff} calculation based on calibrated sediment transport equations is not sensitive to the type of transport equation used in bed load–dominated, coarse bed rivers. This is because the location of peak of the sediment yield density curve (i.e., the discharge value or Q_{eff}) does not shift across different sediment transport relations [Fig. 1(a)]. The discharge value at the location of this peak is a function only of β or the slope of the empirical discharge-sediment transport relation, as well as the width of the discharge bins (Soar and Thorne 2001). The same is true for Q_h because it is based on the relative position on the cumulative sediment yield curve. Errors in absolute sediment yield estimates will not affect this.

However, the definition of bed material load will affect the calculation of Q_h. The present study used suspended sand load as a proxy for bed material load in fine bed rivers. A total bed material load equation used to model this would likely produce similar results because bed load in sand bed rivers is typically a factor of suspended load (Nash 1994; S. Michels-Boyce, "A comparison between rating curves generated from total and suspended bed material loads in sand bed channels," unpublished report). In coarse bed rivers, bed load measurements made by a Helley-Smith sampler were used, which captures coarse gravel to sand-sized material. Suspended sand may be a significant component of total bed material load in some coarse bed rivers (Whiting et al. 1999). Its inclusion in the sediment rating curve would change the magnitude of Q_h relative to Q_{bf}, likely reducing it due to a smaller value of the sediment rating curve exponent. Sediment rating curves based on bed load data from bed load traps, which tend to capture coarse grain sizes and not sand (Bunte and Abt 2009), would also likely result in a different relationship between Q_h and Q_{bf}. Therefore, the correspondence of Q_h with Q_{bf} is dependent on the type of sediment load data used in coarse bed rivers. Estimates of Q_h made from measured and modeled bed material load transport are compared in a separate study.

Average daily flow records were used to calculate sediment yield metrics in this study, which can introduce error in calculating these sediment yield metrics in flashy streams. This error increases with flow regime variability and the value of β (Rosburg 2015). The value of β at a particular site is largely influenced by channel geometry and bed material grain size (Emmett and Wolman 2001; Barry et al. 2004). The present study finds that both Q_{eff} and Q_h tend to overpredict Q_{bf} for values of $\beta > 2.5$ in coarse bed streams (see supplemental material section "Predictor Bias and Sensitivity" and Fig. S1). The sensitivity of Q_h to prediction from semi-empirical sediment transport relations used where sediment measurements are not available will be explored in future study.

The Q_h calculation requires the same data as the Q_{eff} calculation. Biedenharn et al. (2000) and Soar and Thorne (2001) give thorough explanations of data sources and calculation procedures for Q_{eff}. The method used in this research diverges from conventional approaches. General approaches to calculating Q_h given a variety of data sources and availability are as follows:

1. Flow record:
   - Ungauged sites require a scaled regional flow duration curve (FDC) or scaled flow record from a nearby river. A FDC may be scaled using an index discharge (e.g., Biedenharn et al. 2000; Torizzo and Pitlick 2004), or a more complex method may be used (e.g., Fennessey and Vogel 1990). If using a FDC, it must first be converted to a CDF (CDF = 1 − FDC), and then transformed to a PDF through numerical differentiation, as described in "Bankfull Discharge Prediction."
   - Gauged sites require a flow record with at least 10 years of daily flow data. Subdaily flow data (e.g., hourly) are preferred due to the highly nonlinear relationship between flow and sediment transport, but are often difficult to find for longer periods of time.
2. Sediment transport relation:
   - When no sediment transport data are available, a calibrated sediment transport relation appropriate for the river of interest (e.g., total bed material load equation for fine bed rivers and bed load equation for coarse bed rivers) (e.g., Hey 1996; Torizzo and Pitlick 2004).
   - When measured sediment transport data are available, an empirical relation between discharge and sediment transport. For suspended load (>0.0625 mm) and bed load data a bias-corrected linear regression between log-transformed variables can perform well (Vogel et al. 2003; Bunte et al. 2014). *LOADEST* (Runkle et al. 2004), a USGS statistical package, provides other options for multivariate linear regression between suspended sediment load and various flow metrics, which may improve the fit. If using *LOADEST*, it is better to also use add-on software that formats data and outputs for the USGS software such as *LOADRUNNER* (Booth et al. 2007) or Purdue University's online tool (Frankenberger et al. 2011).

Conclusion

The accuracy and bias of two predominant methods used for predicting Q_{bf} are evaluated: (1) Hydrologic predictors based on a peak flood discharge with a specified return interval as an analog to Q_{bf}: the 1.5- to 2-year recurrence interval flood ($Q_{1.5}$ and Q_2, respectively), and (2) process-based predictors based on the magnitude and frequency of sediment transport: the effective discharge, Q_{eff}, and the half-yield discharge, Q_h. Sediment transport data are analyzed concurrent with long-term flow records from 95 sites across the United States ranging from coarse bed, bed load–dominated channels and fine bed, suspended load–dominated channels with drainage areas ranging from 6 km^2 to 1.4×10^5 km^2. It was found that:

1. The half-yield discharge—the discharge associated with 50% of cumulative sediment yield—predicts Q_{bf} well compared to other methods in both coarse and fine bed rivers.
2. When compared to Q_{eff} and the 1.5- and 2-year floods, Q_h has the lowest relative error in predicting Q_{bf} for coarse and fine bed sites.
3. Log-log regression models of observed–predicted data pairs indicate that Q_h and $Q_{1.5}$ calculated for fine bed sites are the only Q_{bf} predictor models whose slopes are not significantly different from unity and whose intercepts are not significantly different from zero.

4. The most effective discharge, Q_{eff}, and Q_h both perform well in predicting Q_{bf} in coarse bed sites, followed by $Q_{1.5}$, whereas Q_{eff} uniformly underpredicts Q_{bf} in fine bed sites.

The behavior of Q_h, a process-based predictor of Q_{bf}, is characterized to highlight circumstances where sediment yield analysis may be important in estimating the bankfull discharge. Guidance is also provided for calculating and using process-based predictors of Q_{bf}. The ability of Q_h to predict Q_{bf} in coarse and fine bed sites represents a novel finding not previously discussed in this context, and an argument for sediment yield analysis to estimate channel design.

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Notation

The following symbols are used in this paper:

m = rank of event (largest = 1);
n = number of events in a sample;
p = statistical significance probability (probability);
Q_{bf} = bankfull discharge (m^3/s);
Q_{eff} = effective discharge (m^3/s);
Q_h = half-yield discharge (m^3/s);
Q_{s25} = discharge at 25% of cumulative sediment yield (m^3/s);
Q_{s75} = discharge at 75% of cumulative sediment yield (m^3/s);
$Q_{1.5}$ = 1.5-year return interval annual flood (m^3/s);
Q_2 = 2-year return interval annual flood (m^3/s);
T = return interval of a flood event (years);
α = significance level (probability) and coefficient for sediment rating curve; and
β = exponent of the sediment rating curve.

Supplemental Data

Tables S1–S3, Eqs. (S1)–(S3), and Figs. S1 and S2 are available online in the ASCE Library (www.ascelibrary.org).

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