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The effect of flow data resolution on sediment yield estimation and channel design

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SUMMARY

The decision to use either daily-averaged or sub-daily streamflow records has the potential to impact the calculation of sediment transport metrics and stream channel design. Using bedload and suspended load sediment transport measurements collected at 138 sites across the United States, we calculated the effective discharge, sediment yield, and half-load discharge using sediment rating curves over long time periods (median record length = 24 years) with both daily-averaged and sub-daily streamflow records. A comparison of sediment transport metrics calculated with both daily-average and sub-daily stream flow data at each site showed that daily-averaged flow data do not adequately represent the magnitude of high stream flows at hydrologically flashy sites. Daily-average stream flow data cause an underestimation of sediment transport and sediment yield (including the half-load discharge) at flashy sites. The degree of underestimation was correlated with the level of flashiness and the exponent of the sediment rating curve. No consistent relationship between the use of either daily-average or sub-daily streamflow data and the resultant effective discharge was found. When used in channel design, computed sediment transport metrics may have errors due to flow data resolution, which can propagate into design slope calculations which, if implemented, could lead to unwanted aggradation or degradation in the design channel. This analysis illustrates the importance of using sub-daily flow data in the calculation of sediment yield in urbanizing or otherwise flashy watersheds. Furthermore, this analysis provides practical charts for estimating and correcting these types of underestimation errors commonly incurred in sediment yield calculations.

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1. Introduction

Current practice in stable channel design focuses on a single "channel forming" discharge that is assumed to be the flow primarily responsible for performing work, transporting sediment, and shaping channel geometry over a period of years (Soar and Thorne, 2001; Doyle et al., 2007). Coupling continuous flow series with sediment-transport relationships to quantify the combined effects of flow and sediment regime using magnitude–frequency analysis (Wolman and Miller, 1960) is increasingly used to compute physically based estimates of channel forming flows, such as the effective discharge (e.g., Shields et al., 2003; Doyle et al., 2007) or the half-load discharge (Sholtes and Bledsoe, 2016).

Sediment yield estimates used in alluvial channel design are calculated from two types of input data: relations between stream

* Corresponding author. *E-mail address:* tyler.rosburg@gmail.com (T.T. Rosburg). discharge and the sediment transport rate, and flow frequency distributions. The accuracy of a sediment yield calculation will therefore be determined by the errors or uncertainties in these two components. The relationship between discharge and sediment transport is often characterized via a sediment rating curve. which is an empirical best-fit power function relating paired instantaneous streamflow and sediment discharge measurements (Walling, 1977). Errors and limitations of sediment rating curves have been extensively studied: for example, they can be imperfect estimators of sediment transport rates due to statistical fitting errors (Ferguson, 1987), storm event hysteresis (Walling, 1977; Moog and Whiting, 1998), non-stationarity in sediment supply over time (Asselman, 2000), and fluctuations inherent to sediment transport (e.g., Kuhnle and Southard, 1988; Curran et al., 2015). Uncertainty in sediment yield calculations due to the flow frequency distribution characteristics, however, has received little attention. In particular, it is not known how the resolution of a streamflow dataset (i.e., daily-averaged vs. sub-daily measurements) can affect sediment yield calculations.







Nomenclature						
a b	sediment rating curve best fit coefficient sediment rating curve best fit exponent, logarithmic	Qs50-Daily	discharge below which 50% of bed material load is transported, computed from daily flow data $\left(\frac{m^3}{s}\right)$			
$D_{50} \\ D_{84} \\ p$	median grain size (mm) 84th percentile grain size (mm) probability of rejecting the null hypothesis when the	Q _{s50-Sub}	discharge below which 50% of bed material load is transported, computed from sub-daily flow data $\left(\frac{m^3}{s}\right)$			
	null hypothesis is true	RB	Richards-Baker flashiness index (Baker et al., 2004)			
Q_{Eff}	effective discharge $\left(\frac{m^3}{s}\right)$	S_{Daily}	channel design slope computed from daily flow			
$Q_{Eff-Daily}$	effective discharge computed with daily-averaged flow data $\left(\frac{m^3}{s}\right)$	S _{Sub}	data channel design slope computed from sub-daily flow data			
Q _{Eff-Sub}	effective discharge computed with sub-daily flow data $\left(\frac{m^3}{s}\right)$	SY SY _{Daily} SVa	sediment yield (m ³) sediment yield computed with daily flow data (m ³) sediment yield computed with sub-daily flow data			
Q _{\$50}	discharge below which 50% of bed material load is transported $\left(\frac{m^3}{s}\right)$	5 I Sub	(m ³)			

The use of daily-averaged streamflow records paired with sediment rating curves to calculate sediment yield implicitly assumes that this resolution of data adequately represents the flow regime. However, studies have shown that small (Ågren et al., 2007), urban (Graf, 1977; Walsh et al., 2005), and arid watersheds (Allan and Castillo, 2007) can exhibit rapid short-term variations in streamflow during runoff events. This type of streamflow behavior is termed "flashy." In flashy watersheds, flows that transport high sediment loads may happen infrequently and for very brief periods of time; in these situations, daily-averaged flow data may not adequately capture the magnitude of discharge most important for sediment transport.

It was recognized long ago that using sediment rating curves with daily-averaged flow data could cause errors in the computation of sediment discharge if the daily-average stream discharge is not representative of the flow rate throughout the day (Colby, 1956). Because sediment discharge is nonlinearly related to stream discharge, small errors in the magnitude of streamflow may cause large errors in the estimation of sediment transport. However, quantitative relationships between flow regime characteristics such as flashiness, characteristics of the sediment transport rating curve, and the relative error in sediment transport metrics due to flow data resolution have not been established. We hypothesize that errors in sediment transport and yield calculations based on daily-average flow data systematically increase with stream flashiness.

There have been few studies exploring how flow data resolution can affect sediment yield calculations. A study of six watersheds in East Devon, England showed that sediment yield calculations from daily flow records could vary by up to 10% from those made with instantaneous records (Walling, 1977). A study of small to medium-size watersheds (smaller than 620 km²) of the Yazoo River basin in Northwest Mississippi revealed that sediment yield curves created from daily-averaged flow data can deviate from 15-min sediment yield curves by more than 100% (Hendon, 1995). This was because the highest discharges, occurring less than 3% of the time, were smoothed out in the daily-averaged data. In another study from the same basin, use of daily-average flow data were found to underpredict sand yield by 51% and total suspended sediment yield by 59% (Dubler, 1997). To the best of our knowledge, the effect of flow data resolution on sediment transport calculations has not been investigated anywhere else.

In this paper, we explore the effect of flow data resolution on sediment transport metrics for both bedload and suspended load transport for fine and coarse bed rivers across the United States. The objectives of this paper are: (i) to quantify the effect of flow data resolution (daily-averaged and sub-daily) on sediment yield calculations; (ii) to investigate the factors such as hydrologic flashiness or sediment rating curve characteristics that most strongly influence the error in calculating sediment yield metrics so that we may better understand under what conditions it is important to have sub-daily flow data, (iii) to provide readers the necessary tools to self-identify situations in which using dailyaveraged flow data for sediment transport calculations is, or is not acceptable based on their own specifications; and (iv) to investigate the potential impacts on channel design parameters when daily-averaged flows are used in situations where sub-daily flows are more appropriate.

2. Methods

2.1. Data selection

This analysis draws on sediment transport data and flow records that were assembled for a related study concerning the magnitude and frequency of sediment transport in U.S. streams and rivers (Sholtes, 2015). Sites used in this analysis have >15 measurements of sediment load and instantaneous discharge collected adjacent to a stream gage with a long term record (median record length = 24 years) of daily and sub-daily discharge data. In total, 39 sites with bedload data and 99 sites with suspended sand load data were included in this analysis (Fig. 1).

The sites cover a wide range of the conterminous United States and represent drainage areas ranging from approximately 10 to 2,500,000 km². Basins were chosen such that a wide range of flow regimes would be analyzed including flashy and non-flashy systems. Summary information for sites used in the present study can be found in Tables S1 and S2 in the supplementary materials.

All bedload data were collected using Helley-Smith bedload samplers as this type of sampler has been the most widely used within the U.S., and the vast majority of existing bedload data were collected with this device (USFS, 2014a,b). Suspended load data were retrieved from the USGS Sediment Data Portal (http://cida. usgs.gov/sediment), an on-line database of suspended sediment measurements for sites across the U.S. Bedload data are used to represent sediment transport in coarse bed rivers with median bed material grain sizes >4 mm (gravel and larger) at sampling sites. Suspended sand load (>0.0625 mm) measurements are used to represent sediment transport in sand bed rivers with median bed material grain sizes ≤1 mm. These sites are referred to as



Fig. 1. Map of sites used in this study.

bedload and suspended load sites based on the data used to represent sediment transport within them. Fine bed sites are scattered geographically and have a range of flow regimes, whereas coarse bed sites are clustered in the U.S. Rocky Mountain and Northwest regions due to lack of concurrent bed load and stream gage data elsewhere. Flow regimes for coarse bed sites are mostly snowmelt dominated, though some flashier systems are represented in the data from the Pacific Coast and Southwest regions of the U.S. (Fig. 1).

Daily flow data and sub-daily flow data (after Oct. 1, 2007) were obtained from USGS National Water Information System (http:// www.waterdata.usgs.gov/nwis) website. Sub-daily flow data (prior to Oct. 1, 2007) were obtained through the USGS Instantaneous Data Archive (http://www.ida.water.usgs.gov/ida). The record length of flow data retrieved varied by site but ranged from the first day in which sub-daily flow data were available through the water year 2013, if possible. However, some gages were discontinued prior to 2013. In total, 80% of sites used in this analysis contained more than 10 years of flow data.

Flow data downloaded from the USGS were filtered prior to analysis. All blank observations and observations of "ice" were removed. Additionally, due to discontinuities in both the daily and sub-daily flow data, the flow data were filtered to ensure that the period of record of both datasets were identical. For some sites, the sub-daily flow data included a mix of observations recorded at 15 min and one hour intervals. To ensure consistent flow data resolution for our analysis, all sub-daily flow data were sampled on the top of the hour to create a consistent set of hourly observations.

2.2. Hydrologic flashiness

We hypothesized that sub-daily flow data would be most accurate in estimating sediment yield for hydrologically flashy systems because daily-averaged flow data do not adequately represent the flow regime. Stream flashiness was calculated with daily-averaged flow data at each site. Daily-averaged flow, as opposed to subdaily flow data, was used to quantify flashiness because dailyaveraged data are the most commonly available streamflow data, and flashiness calculations, therefore, need not be limited to sites with sub-daily flow data. Several methods have been proposed for quantifying stream flashiness (Baker et al., 2004; Konrad and Booth, 2002), and most flashiness metrics are well correlated (Sholtes, 2015). Here we use the Richards–Baker flashiness index (Baker et al., 2004):

$$RB = \frac{\sum_{i=1}^{n} |q_i - q_{i-1}|}{\sum_{i=1}^{n} q_i}.$$
(1)

Here, *RB* is the Richards–Baker flashiness index, q_i is the dailyaveraged discharge on day *i*, and *n* is the total number of days in the flow record. *RB* calculates the ratio of the sum of the absolute values of day-to-day changes in discharge to the sum of mean daily flows. The *RB* index is high for sites characterized by large interdaily variability in discharge.

2.3. Sediment rating curves

Relationships between water discharge and sediment load may often take the form of a simple power function: $Q_s = aQ^b$, where Q_s is the sediment discharge rate (kg/s), Q is the concurrentlymeasured water discharge rate (m³/s), and a and b are best fit regression parameters (Asselman, 2000; Syvitski et al., 2000). This function is referred to as a sediment rating curve. In this relationship, it has been suggested that the exponent b is related to the transport capacity in excess of sediment supply within a river channel, while the coefficient a is related to absolute sediment supply (Barry et al., 2004). The exponent, b, tends to increase with the size of bed material in coarse bed rivers (Emmett and Wolman, 2001).

To estimate the parameters of the sediment rating curve, both the water and sediment transport data were log-transformed as they are heteroscedastic. We fit the log-log models using the rlm() function in the MASS package (Venables and Ripley, 2002) in R (R Core Team, 2014). This is a robust linear regression method that is less sensitive to outliers than ordinary least squares (OLS) regression, and does not require residuals to be normally distributed (though is does require residuals to be symmetric). To correct for transformation bias when un-transforming the parameters, the multiplicative bias correction factor discussed by Ferguson (1987) was used. All log-log slopes (b values) are significant using an approximation of normality test calculated with the lmRob() function in the robust package in R (Wang et al., 2014) (maximum p = 0.019). Multiple R^2 values from these robust linear models are calculated from the weighted residuals from the robust linear models and ranged from 0.25 to 0.79



Fig. 2. Qualitative illustration of sediment transport metrics used in this study.

(median = 0.59); however, 76% of these values are greater than or equal to 0.5. These multiple R^2 values are lower than the conventional R^2 values derived from OLS regression. For comparison, values of R^2 from OLS regression range from 0.22 to 0.96 (median = 0.75) with 94% \ge 0.5.

2.4. Sediment transport metrics

To characterize sediment discharge at each site, three sediment transport metrics were calculated using both the daily-averaged and the sub-daily flow data. The sediment transport metrics utilized in this study are the effective discharge (Q_{Eff}), the discharge below which 50% of the bed material load is transported on average (Q_{s50}), and the sediment yield (*SY*) (Fig. 2).

 Q_{Eff} refers to the increment of discharge that transports the largest fraction of the annual sediment load over a period of years (Andrews, 1980). For some streams and rivers, Q_{Eff} is often considered to be the "channel forming discharge" and nearly equivalent to bankfull discharge (Andrews, 1980; Doyle et al., 2007; Soar and Thorne, 2013). To compute Q_{Eff} , the sediment rating curve (green¹ curve in Fig. 2) is multiplied by the flow frequency curve (blue curve in Fig. 2) to produce a sediment yield curve (red curve in Fig. 2), which represents the average amount of sediment transported by each flow rate over the period of record. Q_{Eff} is the flow corresponding to the peak of the sediment yield curve.

In practice, the effective discharge is generally computed by first subdividing the range of streamflows observed during a period of record into a number of classes or bins from which the total sediment quantity transported by each class is calculated. This is achieved by multiplying the frequency of flow occurrence in each class by the median sediment load for that flow class (Biedenharn et al., 2000) estimated from a sediment rating curve. This procedure results in a series of discrete product values that form an effectiveness curve, with Q_{Eff} being the flow corresponding to the discharge of the flow class with the maximum sediment yield, or peak of this product curve.

The procedure by which the flow record is binned into a histogram to approximate the flow frequency curve can influence the computed Q_{Eff} , Q_{Eff} has been calculated using bins spaced both arithmetically and logarithmically, with most analyses using arithmetic bins (Soar and Thorne, 2001). In this study, we initially divided the flow records at each site into 25 arithmetic bins (e.g., Biedenharn et al., 2000), and this streamflow frequency histogram was multiplied by the sediment rating curve to create a sediment yield curve. If the resulting Q_{Eff} fell into the first bin, the first bin was subdivided into three new bins and the analysis was repeated. This was done to prevent the under-estimation of Q_{Eff} . If Q_{Eff} fell into the first bin again, the original first bin was subdivided into 5 bins. This process was repeated until Q_{Eff} no longer occurred in the first bin or until the original first bin has been subdivided into 11 or more bins. Once the bin with the maximum sediment yield was identified, the median flow in that bin was considered Q_{Eff} .

We performed trapezoidal numerical integration using the trapz function in the PRACMA package (Borcher, 2015) in R on the sediment yield curve (red line, Fig. 2) to compute the total sediment yield (*SY*) for the multi-year period of record. Sediment yield is graphically depicted as the area under the red line in Fig. 2.

Lastly, to calculate the half-load discharge (Q_{s50}), an ordered vector of sediment discharges was created by sorting the water discharges and applying the sediment rating curve. The ordered sediment discharge vector was then cumulatively summed and normalized to a fraction of the cumulative sediment yield. Q_{s50} was then determined by locating the water discharge that corresponded to 50% of the cumulative bed material load transport, Q_{s50} , which can be greater than or less than Q_{Eff} depending on the shape of the sediment yield curve.

2.5. Sediment yield – flashiness relationship

We used quantile regression and multi-variate linear regression to investigate the effect of flow data resolution on sediment yield metrics Q_{Eff} , SY, and Q_{s50} . The response variables in these analyses are the ratios of the sediment transport metrics computed from daily-averaged flow data to those which were computed with sub-daily flow data. Sediment yield estimates made with subdaily flow data are more accurate in general than estimates made with daily-averaged data due to the nonlinear relationship between sediment transport and discharge. Therefore, we consider the sediment yield metrics computed with sub-daily data to be reference values against which metrics computed with daily-averaged flow data can be compared. We refer to the differences between metrics computed using daily and sub-daily flows (expressed as ratios SY_{Daily}/SY_{Sub}, Q_{Eff-Daily}/Q_{Eff-Sub}, and Q_{s50-Daily}/Q_{s50-Sub}) as "errors", although it is important to acknowledge that even metrics computed with sub-daily flow data are not likely to be completely accurate.

2.5.1. Quantile regression

Quantile regression (Koenker and Bassett, 1978) was used to analyze the relationships between our response variables $(SY_{Daily}/SY_{Sub}, Q_{Eff-Daily}/Q_{Eff-Sub}, Q_{s50-Daily}/Q_{s50-Sub})$ and *RB* flashiness. While most regression applications estimate rates of change in the mean of the response variable, quantile regression estimates

 $^{^{1}}$ For interpretation of color in Fig. 2, the reader is referred to the web version of this article.

rates of change for all portions of a probability distribution of the response variable (Cade and Noon, 2003). Quantile regression is especially useful for regression models with heterogeneous variances, as ordinary regression techniques may underestimate, overestimate, or fail to identify real changes in the heterogeneous distribution (Cade and Noon, 2003).

2.5.2. Multi-variate linear regression

Multi-variate linear regression was utilized to model relationships between the response variables and a group of predictor variables and their interactions. The predictor variables included in the analysis were *RB*, average annual precipitation derived from 30-year normals (PRISM Group, 2015), watershed drainage area, median bed sediment size (D_{50}), the 84th-percentile of bed sediment size (D_{84}), the best-fit sediment rating curve exponent (b), and the best-fit sediment rating curve coefficient (a). The best regression subsets using 1 and 2 predictor variables and their interaction were identified using the regsubsets function in the LEAPS package (Lumley and Miller, 2009) in R. Predictor variables identified in each best subset were checked for cross-correlation before being regressed, and a maximum R^2 value of 0.20 was allowed amongst variables in a regression model.

3. Results

3.1. Effective discharge

The hydrologic flashiness at a river station as quantified by *RB* did not systematically influence the ratio of effective discharge computed with daily-averaged flow data ($Q_{Eff-Daily}$) to that computed with sub-daily flow data ($Q_{Eff-Daily}$) (Fig. 3). Of the bedload sites, approximately 60% had a $Q_{Eff-Daily}/Q_{Eff-Sub}$ ratio that was larger than unity. Similarly, of the suspended load sites, 40% had a $Q_{Eff-Daily}/Q_{Eff-Sub}$ ratio that was larger than unity. The ratio of $Q_{Eff-Daily}/Q_{Eff-Sub}$ tended to be greater than unity for flashy bedload sites (*RB* > 0.3). Flashiness also appeared to be related to error in $Q_{Eff-Daily}/Q_{Eff-Sub}$ form unity increased. Because no strong relationship between *RB* and $Q_{Eff-Daily}/Q_{Eff-Sub}$ was observed, we proceeded with additional regression analyses only with the other sediment yield metrics: SY_{Daily}/SY_{Sub} and $Q_{s50-Daily}/Q_{s50-Sub}$.

3.2. Quantile regression

3.2.1. Sediment yield

Sediment yield computed with daily-averaged flow data (SY_{Daily}) was found to generally be less than sediment yield computed with sub-daily flow (SY_{Sub}) (Fig. 4). The ratio of SY_{Daily}/SY_{Sub} was found to decrease with flashiness in a wedge-shaped fashion for both bedload and suspended load sites. That is, for non-flashy sites ($RB \approx 0.1$), SY_{Daily}/SY_{Sub} was nearly equal to unity while for flashy sites (RB > 0.4), SY_{Daily}/SY_{Sub} was found to range from 0.4 to 1. Quantile regression was used to further investigate the relationship of SY_{Daily}/SY_{Sub} to flashiness. The various "quantile" lines represent the relationships of the 5th, 25th, 50th, 75th, and 95th percentiles of the SY_{Daily}/SY_{Sub} distribution to flashiness (Fig. 4).

Increased variance in the regression of SY_{Daily}/SY_{Sub} and $Q_{s50-Daily}/Q_{s50-Sub}$ with *RB* flashiness index was observed for sites with a greater magnitude of the sediment rating curve parameter *b*. For the flashiest suspended load sites with *RB* > 0.6, as *b* increases, the ratio of SY_{Daily}/SY_{Sub} decreases (Fig. 5). There is not a strong correlation between *b* and $Q_{s50-Daily}/Q_{s50-Sub}$ ($R^2 = 0.17$).

3.2.2. Half-load discharge (Q_{s50})

The value of the half-load discharge calculated with daily flow data ($Q_{s50-Daily}$) was generally less than its sub-daily flow counterpart ($Q_{s50-Sub}$) (Fig. 6). Much like the SY data, the Q_{s50} data formed a wedge-shaped pattern in which the sediment rating curve parameter *b* influenced the degree of response of $Q_{s50-Daily}/Q_{s50-Sub}$. That is, for non-flashy sites ($RB \approx 0.1$), $Q_{s50-Daily}/Q_{s50-Sub}$ was nearly equal to unity, while for flashy sites (RB > 0.4), $Q_{s50-Daily}/Q_{s50-Sub}$ ranged from 0.2 to 1. Just as we observed in the SY_{Daily}/SY_{Sub} results, the sediment rating curve parameter *b* contributed to the degree of response of $Q_{s50-Daily}/Q_{s50-Sub}$ to flashiness. For sites with flashy flow characteristics, as rating curve parameter *b* increased, the ratio of $Q_{s50-Daily}/Q_{s50-Sub}$ decreased (Fig. 5).

3.3. Multi-variate linear regression analysis

Table 1 presents the best predictors of change in SY_{Daily}/SY_{Sub} and $Q_{s50-Daily}/Q_{s50-Sub}$ as determined from the best subsets analysis of the multi-variate linear regression procedure. *RB* was found to be the best single predictor of change in both in SY_{Daily}/SY_{Sub} and $Q_{s50-Daily}/Q_{s50-Sub}$ for both bedload and suspended load sites as it



Fig. 3. Ratio of effective discharge computed with daily-averaged flow to sub-daily flow vs. the Richards-Baker flashiness index: (a) bedload sites and (b) suspended load sites.



Fig. 4. Ratio of sediment yield computed with daily-averaged flow to sub-daily flow vs. the Richards–Baker flashiness index: (a) bedload sites and (b) suspended load sites. Data points categorized by *b* (sediment rating curve exponent).



Fig. 5. Relationship between the underestimation of SY, Q_{s50} , and the sediment rating curve best fit exponent, *b*, for suspended load sites with a *RB* flashiness greater than 0.6.

explained more variance than any other single variable (i.e., higher R^2 value). The second- and third-best indicators for all models were b and D_{50} . The model with the highest adjusted R^2 was a 2-variable interaction model in which *RB* and *b* are the independent variables. All models in Table 1 are significant at p < 0.0001.

4. Discussion

4.1. Effective discharge

The decision to use daily-averaged or sub-daily streamflow data was not found to impact the calculation of effective discharge in a consistent way. Amongst bedload and suspended load sites, the ratio of $Q_{Eff-Daily}/Q_{Eff-Sub}$ varied around unity with nearly equal abundance. These results are a byproduct of the sensitivity of the Q_{Eff} calculation to the discretization of the streamflow record as well as the transience of flashy flows, which in some but not all cases influence the shape of the flow-frequency distribution when comparing a histogram of flows made from daily-average vs. sub-daily flow records.



Fig. 6. Ratio of discharge below which 50% of bed material load is transported computed with daily-averaged flow to sub-daily flow vs. the Richards–Baker flashiness index: (a) bedload sites and (b) suspended load sites. Data points categorized by *b* (sediment rating curve exponent).

Bedload/suspended load	Dependent variable	Number of independent variables	Best regression model	Adj. R ²
Bedload	SY _{Daily} /SY _{Sub}	1	1.02–0.45 RB	0.37
Bedload	$Q_{s50-Daily}/Q_{s50-Sub}$	1	1.03–0.55 RB	0.38
Suspended load	SY _{Daily} /SY _{Sub}	1	1.05–0.41 RB	0.33
Suspended load	$Q_{s50-Daily}/Q_{s50-Sub}$	1	1.04–0.50 RB	0.48
Bedload	SY _{Daily} /SY _{Sub}	2	1.11–0.55 RB – 0.03 b	0.51
Bedload	$Q_{s50-Dailv}/Q_{s50-Sub}$	2	1.12–0.65 RB – 0.03 b	0.46
Suspended load	SY _{Daily} /SY _{Sub}	2	1.18–0.41 RB – 0.07 b	0.37
Suspended load	$Q_{s50-Daily}/Q_{s50-Sub}$	2	1.12 – 0.50 RB – 0.04 b	0.49
Bedload	SY _{Daily} /SY _{Sub}	2 + interaction	0.95 + 0.63 RB + 0.04 b - 0.57RB [*] b	0.82
Bedload	$Q_{s50-Daily}/Q_{s50-Sub}$	2 + interaction	0.94 + 0.66RB + 0.05 b - 0.63RB [] b	0.72
Suspended load	SY _{Daily} /SY _{Sub}	2 + interaction	1.01 + 0.38 RB + 0.02 b - 0.43RB [*] b	0.46
*Suspended load	$Q_{s50-Daily}/Q_{s50-Sub}$	2 + interaction	$0.99 + 0.10RB + 0.03 \ b - 0.32 \ RB^{\circ}b$	0.53

Table 1Multi-variate linear regression models for the prediction of SY_{Daily}/SY_{Sub} and $Q_{s50-Daily}/Q_{s50-Sub}$ for suspended and bedload sites.

RB = Richards-Baker flashiness index computed with daily-average flow data.

b = Best fit exponent from sediment rating curve.

* Used in the development of Fig. 11.

For ratios of $Q_{Eff-Daily}/Q_{Eff-Sub}$ less than unity, differences in the size of the bins (range of discharges) in the discretized flow duration curve histogram were the primary cause for the differences in Q_{Eff} . We refer to this as a Type A error. Type A errors were caused by the sub-daily flow record nearly always having a larger maximum flow and smaller minimum flow than the daily-averaged flow record. This greater range between the maximum and minimum flow forces the range of flows in each bin to be greater for sub-daily flow data than daily-averaged data, when a fixed number of arithmetically sized bins is used. Because Q_{Eff} was calculated as

the median discharge of the bin that produces the maximum sediment yield, $Q_{Eff-Sub}$ was likely to be greater than $Q_{Eff-Daily}$ when the same bin was identified as containing Q_{Eff} . This type of error is depicted in Fig. 7.

In the case of $Q_{Eff-Daily}/Q_{Eff-Sub}$ values that are greater than unity, differences in flow frequency were found to be the primary cause of this type of error, which we termed Type B error. After being multiplied by the sediment rating curve, differences in the frequency of high magnitude discharges between the sub-daily and daily-averaged flow records often caused Q_{Eff} to be located in



Fig. 7. Type A error for the Trinity River near Hoopa, CA. Differences in bin sizes cause disparity in Q_{eff}, even when the same bin is identified as most effective.

different bins, thereby causing larger departures in Q_{Eff} . At high discharges, these differences in flow frequency could be very small and still cause large differences in Q_{Eff} when the sediment transport rate at that discharge was particularly high. This was a common result for flashy (RB > 0.3) bedload sites. An example of this type of error is shown in Fig. 8, where the arrows in the discharge probability density histograms indicate the effective discharge computed with daily-averaged flows. The sub-daily flow record captures many more high flows and thus produces a smoother histogram and effectiveness curve, which causes the effective discharge bin computed with sub-daily flows to differ dramatically from that computed with daily-averaged flows.

As flashiness increases, the sub-daily flow record departs from the daily-average flow record. This causes Type A and Type B errors to become more prevalent and of greater magnitude. Fig. 3 depicts how the departure of $Q_{Eff-Daily}/Q_{Eff-Sub}$ from unity can increase with flashiness. These results indicate that Q_{Eff} is very sensitive to the binning method used, the size of individual bins, and slight differences in flow frequency distributions between daily and sub-daily flow data. Comparatively, Q_{s50} does not suffer from the problems associated with representing the flow frequency distribution, and was found to be much more consistent in magnitude than Q_{Eff} . However, both Q_{Eff} and Q_{s50} are sensitive to the slope of the sediment rating curve, *b*, which may be difficult to accurately estimate with sparse sediment transport data, though previous research suggests that *b* can be accurately estimated for coarse bed sites (Barry et al., 2008). Additionally, recent work suggests that Q_{s50} may perform better than Q_{Eff} in the prediction of bankfull discharge for fine bed streams (Sholtes and Bledsoe, 2016). Therefore, for certain applications such as channel design, Q_{s50} may be a more useful metric than Q_{Eff} .

4.2. Estimating error in sediment yield and half-load discharge calculations

Using the 2-variable interaction model developed through multi-variate linear regression (Table 1), we can quantitatively describe the differences in *SY* and Q_{s50} values resulting from daily-averaged versus sub-daily flow data. Figs. 9 and 10 present the percent disagreement between sub-daily and daily values of *SY* and Q_{s50} as a function of *RB* and *b*.

Because estimating disagreement in SY and Q_{s50} computed with daily-averaged flow data using Figs. 9 and 10 requires *b*, it is important to have a reliable method for estimating *b*. The logarithmic slope of the sediment transport function, *b*, is thought to represent a number of physical watershed characteristics including basin area, the erosive power of the river, and the extent to which new sediment sources become available as discharge increases (Asselman, 2000; Syvitski et al., 2000; Bunte et al., 2015). The parameter *b* can be most reliably estimated by using a series of sediment discharge measurements and corresponding flow rates to fit a power law function to these data; however, these measurements are often not available and it can be cost-prohibitive to collect new sediment discharge measurements. In this case, one may



Fig. 8. Type B error for the Mad River near Arcata, CA (*RB* = 0.299). Differences in discharge density (indicated by dashed arrows) caused the effective discharge to be located in different bins.



Fig. 9. Percent difference in sediment yield (SY) (values labeled at the top of contours) calculated with daily-averaged flow data: (a) bedload sites and (b) suspended load sites.



Fig. 10. Percent difference in half-load discharge (Q_{s50}) (values labeled at the top of contours) calculated with daily-averaged flow data: (a) bedload sites and (b) suspended load sites.

use channel geometry and slope measurements with a total bed material load equation (e.g., Bagnold, 1966; Einstein, 1950; Yang, 1973; Brownlie, 1981) to estimate sediment transport as a function of discharge, from which *b* can be determined. If local channel geometry data are also unavailable, one may have to rely on a regression equation in order to estimate *b*. In this circumstance, regression equations developed for localized conditions are generally best, but when these are not available, more global regression equations may be carefully considered (Syvitski and Morehead, 1999; Bunte and Abt, 2003).

4.3. Implications for design of channel bed slope

Sediment yield calculations for flashy systems (RB > 0.4) with a moderate to large sediment rating curve exponent (b > 2) using daily-averaged flow data are at risk of greatly underestimating sediment yield. To quantify the potential effect of underestimating sediment yield on channel design, we use a stable channel proportionality (Henderson, 1966) as a first approximation to explore how channel design slope may differ when using daily-averaged or sub-daily flow data. Henderson (1966) combined the Einstein sediment transport function as revised by Brown (1950), the Chezy flow resistance formula, and momentum and mass conservation for steady uniform flow into a single proportionality where q_s is the unit sediment transport rate, q is the unit water discharge, Sis the channel slope, and D is the grain size:

$$q_s \propto \frac{q^2 S^2}{D^{\frac{3}{2}}} \tag{2}$$

Rearranging for channel slope yields:

$$S \propto \sqrt{\frac{D^2 q_s}{q^2}}$$
 (3)

Comparing a slope resulting from daily-averaged discharges to one resulting from sub-daily discharges yields:

$$\frac{S_{Daily}}{S_{Sub}} \propto \frac{\sqrt{\frac{D^2}{q^2_{s-Daily}}}}{\sqrt{\frac{D^2}{q^2_{sub}}}}$$
(4)

Wilcock (2004) suggested that the generic form of the relationship in Eq. (4) could be used to check for the potential aggradation or degradation of a channel. A ratio of $\frac{S_{Daily}}{S_{Sub}}$ that is less than unity suggests that use of daily-averaged flow data may result in a bed slope (S_{Daily}) that is too low, thus creating potential for channel aggradation. Likewise, if the ratio of $\frac{S_{Daily}}{S_{Sub}}$ is greater than unity, S_{Daily} may be overestimated, creating potential for channel degradation.

For our purposes, we assume that bed sediment size remains constant and simplify Eq. (4), yielding:

$$\frac{S_{Daily}}{S_{Sub}} \propto \sqrt{\frac{q_{s-Daily}}{q_{s-Sub}}} * \left(\frac{q_{Daily}}{q_{Sub}}\right)^{-1}$$
(5)

Replacing q_s with a sediment rating curve $(q_s = aq^b)$ yields:

$$\frac{S_{Daily}}{S_{Sub}} \propto \sqrt{\frac{aq_{Daily}^b}{aq_{Sub}^b}} \left(\frac{q_{Daily}}{q_{Sub}}\right)^{-1} = \left(\frac{q_{Daily}}{q_{Sub}}\right)^{\frac{b}{2}-1}$$
(6)

Evaluation of (6) requires an estimate of a single dominant discharge, computed with daily-averaged and sub-daily flow data. Sholtes and Bledsoe (2016) have shown that Q_{s50} is a useful process-based predictor of bankfull discharge, so if we use Q_{s50} as our measure of discharge, we arrive at:

$$\frac{S_{Daily}}{S_{Sub}} \propto \left(\frac{Q_{s50\text{-}Daily}}{Q_{s50\text{-}Sub}}\right)^{\frac{D}{2}-1}$$
(7)

The ratio of $Q_{s50-Daily}/Q_{s50-Sub}$ in Eq. (7) can be estimated for both bedload and suspended load sites using the two variable plus interaction relationships presented in Table 1 (denoted with an asterisk in column 1). We can then use Eq. (7) to plot the amount by which a design slope calculated using daily-average flow data, S_{Daily}, differs from the design slope calculated with sub-daily flow data, S_{sub}. Presumably the slope calculated such that sediment continuity is achieved based on sediment yield estimates made from sub-daily flow data (S_{Sub}) should be closer to a true equilibrium slope (that is, more accurate). Fig. 11 shows that for both bedload and suspended load sites, high flashiness and high b values correspond to low values of $\frac{S_{Daily}}{S_{Sub}}$. This indicates that using daily-averaged flow data in these types of conditions would result in a channel design slope that is greatly underestimated, which could lead to channel aggradation. Conversely, for both suspended load and bedload sites there exist certain combinations of flashiness and b that cause $\frac{S_{Daily}}{S_{Daily}}$ to exceed 1. These are areas with a RB flashiness less than 0.1 and a

b greater than 4, an unlikely combination. For suspended load sites, these are also areas with a *RB* flashiness greater than 0.8 and *b* near 1, also unlikely. In these cases, daily-averaged flow data causes an over-estimation of channel design slope, which could potentially cause channel degradation.

5. Conclusions

Our analysis of the effects of flow data resolution on sediment transport calculations for 39 bed load sites and 99 suspended load sites suggests that the use of daily-averaged flow data is not always appropriate because it can result in a substantial underestimation of sediment yield metrics if stream flashiness or the exponent of the sediment rating curve are high (RB > 0.4)or b > 2). The effect of flow data resolution on the computation of effective discharge, Q_{Eff}, was not readily predictable as a function of stream flashiness index, RB, or sediment rating curve exponent, b, because Q_{Eff} is highly sensitive to the procedure used represent the flow frequency distribution. Deviations of values of the half-load discharge, Q_{s50}, and the sediment yield, SY, calculated with daily vs. sub-daily streamflow data correlate well with stream flashiness RB and the rating curve exponent b. This is understandable from a theoretical standpoint, as sediment transporting flows are more likely to be missed or underestimated by daily-average flow data when a stream is flashy, and the impact of the error is greater when the sediment rating curve is steep. In instances where the flow regime at a site is flashy (RB > 0.4)and the sediment rating curve exponent is greater than two (b > 2), SY and Q_{s50} were greatly underestimated using dailyaveraged flow data. This underestimation of SY and Q_{s50} may then lead to underestimation of channel design slope, which could ultimately cause channel aggradation (Fig. 11). Additionally, even in instances where flashiness was high, low logarithmic sediment rating curve slope $(b \approx 1)$ controlled the magnitude of error in sediment yield and Q_{s50} calculations, causing error to be quite small. Based on these results, it is better to use sub-daily discharge data for sediment transport calculations when the site either has a RB flashiness greater than 0.4 or b is greater than 2. This should help keep the error of sediment transport metrics (SY, Q_{s50}) and channel design slope computed with dailyaveraged flow data to within 20% when compared to the same metrics computed with sub-daily flow data.



Fig. 11. Ratio of design slope calculated with daily flow data (*S*_{Daily}) to the design slope calculated with sub-daily flow data (*S*_{Sub}): (a) bedload sites and (b) suspended load sites.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.jhydrol.2016.04. 040.

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